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HEAT AND MASS TRANSFER IN POROUS AND DISPERSE MEDIA

PLANE-PARALLEL NONISOTHERMAL FILTRATION OF A GAS: THE ROLE OF HEAT TRANSFER

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The influence of the parameters of a mathematical model and of the type of boundary conditions on the dynamics of pressure and temperature fields in nonisothermal gas filtration has been investigated in a computational experiment. To describe the process, the nonlinear system of partial differential equations obtained from the mass and energy conservation laws and Darcy law were used, with the physical and caloric equations of state employed as closing relations. The boundary conditions correspond to gas injection at a given mass flow rate of different intensities.

Keywords: *nonisothermal filtration, imperfect gas, heat conduction, convective transfer, finite-difference methods.*

Introduction. Up to the present time, the calculations needed for planning the development and exploitation of gas and gas-condensate deposits have been based either on the use of mathematical models of a perfect gas, or on the introduction of some corrections for the gas imperfection, but with the averaging of the corresponding thermodynamic functions (imperfection and throttling coefficients) in the entire range of changes in pressure and temperature. It is evident that such an approach not only has a limited application, but also is methodologically inapplicable, since it lacks scientific justification and cannot be systematized. From the application point of view, it is also not justified, since it is incompatible with the current tendencies of involving, into development, gas deposits located at large depths, i.e., having a high pressure and temperature. In the present paper, using as an example plane-parallel gas flows in a porous medium, an analysis of the mutual effect of the thermodynamics and field of the rate of filtration, as well as of the input parameters of a mathematical model in terms of boundary and initial conditions, has been made. The basic hydrodynamic and thermodynamic characteristics of the process of nonstationary filtration of a perfect (ideal) gas have been studied in [1]; nevertheless in the present article we suggest the results of a comparable analysis of the behavior of a perfect and imperfect gases. The analysis is made for the problem of injection rather than suction, since in the latter case, the specification of a boundary condition corresponding to the known temperature is lacking natural justification.

Statement of the Problem. For mathematical description of the process of gas injection into a thermally insulated bed through a linear gallery, we will avail ourselves of the full system of equations that describe the plane-parallel nonisothermal filtration of an imperfect gas:

$$\frac{\partial}{\partial t}\left(\frac{p}{zT}\right)=\frac{\partial}{\partial x}\left(\frac{p}{zT}\frac{\partial p}{\partial x}\right),$$

$$\frac{\partial T}{\partial t}=\delta\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\left(1+\frac{T}{z}\frac{\partial z}{\partial T}\right)\frac{\partial p}{\partial t}+\frac{c_p}{R}\frac{p}{zT}\frac{\partial T}{\partial x}\frac{\partial p}{\partial x}-\frac{T}{z}\frac{\partial z}{\partial T}\left(\frac{\partial p}{\partial x}\right)^2,\quad 0<x<1,\quad t>0, \tag{1}$$

where $\overline{T} = c_{\rm r}T/(mp_0)$, $\overline{p} = p/p_0$, $\overline{x} = x/l$, $\delta = \kappa/\kappa_p$, $\kappa_p = kp_0/(m\mu)$, and $\overline{t} = \kappa_p t/l^2$. In Eqs. (1) and in what follows, the bar over dimensionless variables is omitted for the sake of convenience.

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On the inner boundary of the bed we assign a constant gas flow rate $\left(A = \dfrac{m\mu RM}{kHp_0c_{\rm r}}\right)$:

$$-\frac{p}{zT}\frac{\partial p}{\partial x} = A\,, \quad x = 0\,. \tag{2}$$

This condition is supplemented with the condition of constancy of the injected gas temperature, with the condition of bed impermeability for the filtering gas and heat insulation being given on the outer boundary:

$$T = T_{\rm w}\,, \quad x = 0\,; \tag{3}$$

$$\frac{\partial p}{\partial x} = 0\,, \quad \frac{\partial T}{\partial x} = 0\,, \quad x = 1\,. \tag{4}$$

At the initial moment the pressure and temperature are considered constant:

$$p\,(x, 0) = 1\,, \quad T\,(x, 0) = T_0\,, \quad 0 \le x \le 1\,. \tag{5}$$

The following equation is taken as the equation of state [2]:

$$z = \left(0.4\,\ln\left(\frac{mp_0}{c_{\rm r}T_{\rm c}}\,T\right) + 0.73\right)^{\frac{p_0}{p_{\rm c}}p} + 0.1\,\frac{p_0}{p_{\rm c}}\,p\,, \tag{6}$$

where $T_{\rm c}$ and $p_{\rm c}$ are the critical temperature and pressure of the natural gas, which is a mixture of gases mainly of the paraffin series beginning from methane.

Thus, the solution of the initial boundary-value problem (1)–(6) depends on two parameters, δ and c_p/R, that enter into Eq. (1), on boundary condition (2) determined by the dimensionless mass flow rate A, and on two dimensionless complexes $mp_0/(c_{\rm f}T_{\rm c})$ and $p_0/p_{\rm c}$ that enter into the equation of state.

To solve the initial boundary-value problems (1)–(6), we approximate the equations of system (1) by a purely implicit absolutely stable difference scheme obtained with the aid of the balance method:

$$\left(\frac{\overset{j+1}{\overset{s}{y}}_i}{\overset{j+1}{\overset{s}{z}}_i\,\overset{j+1}{\overset{s}{u}}_i} - \frac{\overset{j}{\overset{s}{y}}_i}{\overset{j}{\overset{s}{z}}_i\,\overset{j}{\overset{s}{u}}_i}\right)\Big/\tau = \overset{s}{k}_{i+1}\,\frac{\overset{j+1}{\overset{s}{y}}_{i+1} - \overset{j+1}{\overset{s}{y}}_i}{h^2} - \overset{s}{k}_i\,\frac{\overset{j+1}{\overset{s}{y}}_i - \overset{j+1}{\overset{s}{y}}_{i-1}}{h^2}\,, \quad i = \overline{1, n-1}\,, \quad j = \overline{0, j_0-1}\,;$$

$$\left(\frac{\overset{j+1}{\overset{s}{u}}_i - \overset{j}{\overset{s}{u}}_i}{\tau} - \overset{s}{c}_i\,\frac{\overset{j+1}{\overset{s}{y}}_i - \overset{j}{\overset{s}{y}}_i}{\tau}\right) = \overset{s}{a}_{i+1}\,\frac{\overset{j+1}{\overset{s}{u}}_{i+1} - \overset{j+1}{\overset{s}{u}}_i}{h^2} - \overset{s}{a}_i\,\frac{\overset{j+1}{\overset{s}{u}}_i - \overset{j+1}{\overset{s}{u}}_{i-1}}{h^2} + \overset{s}{b}_i\,\frac{\overset{j+1}{\overset{s}{u}}_{i+1} - \overset{j+1}{\overset{s}{u}}_{i-1}}{2h} - \overset{s}{d}_i\,\overset{j+1}{\overset{s}{u}}_i\,. \tag{7}$$

Here the grid functions corresponding to the unknown functions $p(x,\ t)$ and $T(x,\ t)$ are denoted as y_i^j and u_i^j.

The difference analogs of boundary conditions (2) and (4) were written in the second order of approximation. To obtain a difference scheme for the inner boundary ($i = 0$) at a constant gas flow rate, we integrate the first equation of system (1) in an elementary cell. Then for a constant gas flow rate the difference analog of boundary condition (2) has the form

$$\left(\left(\frac{\overset{j+1}{\overset{s}{y}}_0}{\overset{j+1}{\overset{s}{z}}_0\,\overset{j+1}{\overset{s}{u}}_0} - \frac{\overset{j}{\overset{s}{y}}_0}{\overset{j}{\overset{s}{z}}_0\,\overset{j}{\overset{s}{u}}_0}\right)\Big/\tau\right)\frac{h}{2} = \overset{s}{k}_1\,\frac{\overset{j+1}{\overset{s}{y}}_1 - \overset{j+1}{\overset{s}{y}}_0}{h} + A\,, \quad j = \overline{0, j_0-1}\,. \tag{8}$$

For the temperature at the face of the well

$$\overset{s}{u}{}_0^{j+1} = T_{\text{w}}\,, \quad j = \overline{0, j_0 - 1}\,. \tag{9}$$

The difference approximation of the conditions at the outer boundary ($i = n$) is

$$\left(\left(\frac{\overset{s}{y}{}_n^{j+1}}{\overset{s}{z}{}_n^{j+1}\,\overset{s}{u}{}_n^{j+1}} - \frac{\overset{s}{y}{}_n^{j}}{\overset{s}{z}{}_n^{j}\,\overset{s}{u}{}_n^{j}}\right)\Big/\tau\right)\frac{h}{2} = -\,k_n^s\,\frac{\overset{s}{y}{}_n^{j+1} - \overset{s}{y}{}_{n-1}^{j+1}}{h}\,,$$

$$\left(\frac{\overset{s}{u}{}_n^{j+1} - \overset{s}{u}{}_n^{j}}{\tau} - c_n^s\,\frac{\overset{s}{y}{}_n^{j+1} - \overset{s}{y}{}_n^{j}}{\tau}\right)\frac{h}{2} = -\,a_n^s\,\frac{\overset{s}{u}{}_n^{j+1} - \overset{s}{u}{}_{n-1}^{j+1}}{h} + b_n^s\,\frac{\overset{s}{u}{}_n^{j+1} - \overset{s}{u}{}_{n-1}^{j+1}}{2} - \frac{h}{2}\,d_n^s\,\overset{s}{u}{}_n^{j+1}\,, \quad j = \overline{0, j_0 - 1}\,. \tag{10}$$

We approximate the initial conditions in the form

$$\overset{s}{y}{}_i^{0} = 1\,, \quad \overset{s}{u}{}_i^{0} = T_0\,, \quad i = \overline{0, n}\,. \tag{11}$$

In Eqs. (7)–(10) we put

$$k_i^s = \frac{\overset{s}{y}{}_{i-1/2}^{j+1}}{\overset{s}{z}{}_{i-1/2}^{j+1}\,\overset{s}{u}{}_{i-1/2}^{j+1}}\,, \quad \overset{s}{y}{}_{i-1/2}^{j+1} = \frac{\overset{s}{y}{}_{i-1}^{j+1} + \overset{s}{y}{}_i^{j+1}}{2}\,, \quad \overset{s}{u}{}_{i-1/2}^{j+1} = \frac{\overset{s}{u}{}_{i-1}^{j+1} + \overset{s}{u}{}_i^{j+1}}{2}\,, \quad a_i^s = \delta\,,$$

$$b_i^s = \frac{c_p}{R}\,\frac{\overset{s+1}{y}{}_i^{j+1}}{\overset{s}{z}{}_i^{j+1}\,\overset{s}{u}{}_i^{j+1}}\,\frac{\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}}{2h}\,,$$

$$c_i^s = 1 + \frac{\overset{s}{u}{}_i^{j+1}}{\overset{s}{z}{}_i^{j+1}}\left(\frac{\partial^s z}{\partial T}\right)_i^{j+1}, \quad d_i^s = \frac{1}{\overset{s}{z}{}_i^{j+1}}\left(\frac{\partial^s z}{\partial T}\right)_i^{j+1}\left(\frac{\overset{s}{y}{}_{i+1}^{j+1} - \overset{s}{y}{}_{i-1}^{j+1}}{2h}\right)^2, \quad b_n^s = \frac{c_p}{R}\,\frac{\overset{s+1}{y}{}_n^{j+1}}{\overset{s}{z}{}_n^{j+1}\,\overset{s}{u}{}_n^{j+1}}\,\frac{\overset{s+1}{y}{}_n^{j+1} - \overset{s+1}{y}{}_{n-1}^{j+1}}{h}\,,$$

$$d_n^s = \frac{1}{\overset{s}{z}{}_n^{j+1}}\left(\frac{\partial^s z}{\partial T}\right)_n^{j+1}\left(\frac{\overset{s}{y}{}_n^{j+1} - \overset{s}{y}{}_{n-1}^{j+1}}{h}\right)^2.$$

For the numerical implementation of the difference problem (7)–(11), we shall avail ourselves of the method of simple iterations. We will organize the iteration process as follows:

a) assign the initial approximation $\overset{0}{y}{}_i^{j+1} = y_i^j$, $\overset{0}{u}{}_i^{j+1} = u_i^j$, $i = 0,\ n$;

b) using the pivotal method, solve the linear system of equations for the unknowns $\overset{s+1}{y}{}_i^{j+1}$, $i = \overline{0, n}$:

$$-\,C_0\overset{s+1}{y}{}_0^{j+1} + B_0\overset{s+1}{y}{}_1^{j+1} = -\,F_0; \quad A_i\overset{s+1}{y}{}_{i-1}^{j+1} - C_i\overset{s+1}{y}{}_i^{j+1} + B_i\overset{s+1}{y}{}_{i+1}^{j+1} = -\,F_i\,; \quad i = \overline{1, n-1}\,;$$

$$A_n\overset{s+1}{y}{}_{n-1}^{j+1} - C_n\overset{s+1}{y}{}_n^{j+1} = -\,F_n\,;$$

where

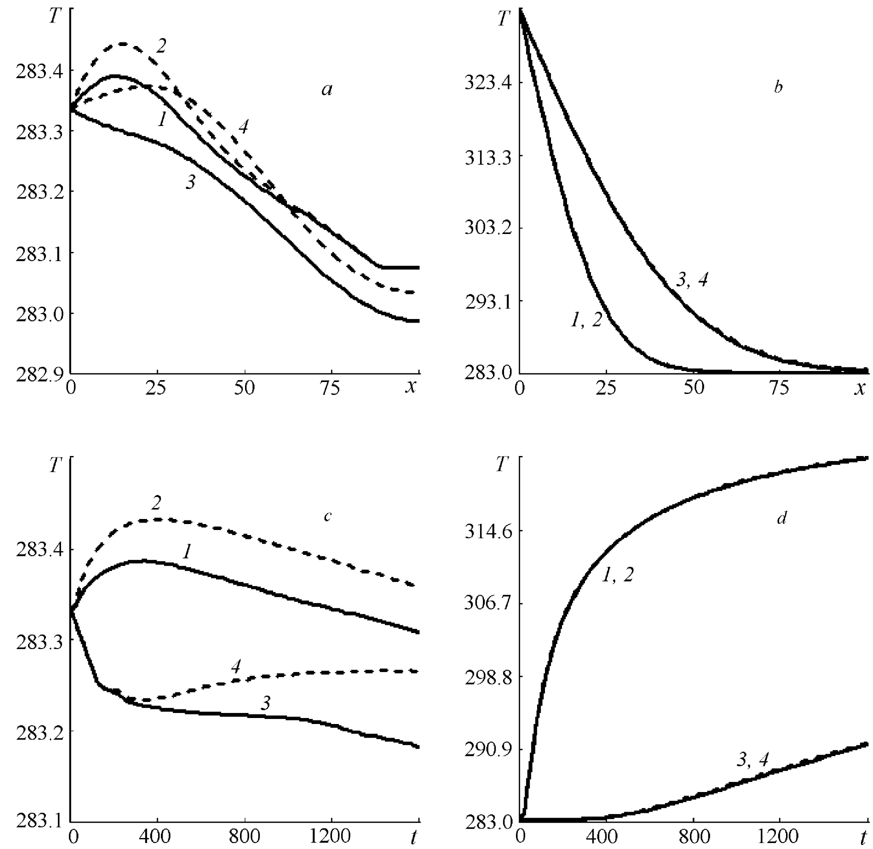

Fig. 1. Distribution (a, b) and dynamics of change (c, d) in the gas temperature at a small flow rate for the first (a, c) and second (b, d) variants of calculation (1, 3) perfect gas; 2, 4) imperfect gas); a, b: 1, 2), t = 400; 3, 4) t = 1600; c, d: 1, 2) x = 10 m; 3, 4) x = 50 m. T, K; x, m.

$$C_0 = \frac{h}{2\tau \overset{s}{z}{}_0^{j+1} \overset{s}{u}{}_0^{j+1}} + \frac{\overset{s}{k}_1}{h}, \quad B_0 = \frac{\overset{s}{k}_1}{h}, \quad F_0 = \frac{\overset{s}{y}{}_0^{j}}{\overset{s}{z}{}_0^{j} \overset{s}{u}{}_0^{j}} \frac{h}{2\tau} + A, \quad A_i = \frac{\overset{s}{k}_i}{h^2}, \quad C_i = \frac{1}{\tau \overset{s}{z}{}_i^{j+1} \overset{s}{u}{}_i^{j+1}} + \frac{\overset{s}{k}_{i+1}}{h^2} + \frac{\overset{s}{k}_i}{h^2},$$

$$B_i = \frac{\overset{s}{k}_{i+1}}{h^2}, \quad F_i = \frac{\overset{s}{y}{}_i^{j}}{\tau \overset{s}{z}{}_i^{j} \overset{s}{u}{}_i^{j}}, \quad A_n = \frac{\overset{s}{k}_n}{h}, \quad C_n = \frac{h}{2\tau \overset{s}{z}{}_n^{j+1} \overset{s}{u}{}_n^{j+1}} + \frac{\overset{s}{k}_n}{h}, \quad F_n = \frac{\overset{s}{y}{}_n^{j}}{\overset{s}{z}{}_n^{j} \overset{s}{u}{}_n^{j}} \frac{h}{2\tau};$$

c) analogously, by the pivotal method we solve the difference equations for the temperature;

d) check the fulfillment of the conditions for the convergence of iterations: $\max\limits_{i=0,n} \left| \overset{s+1}{y}{}_i^{j+1} - \overset{s}{y}{}_i^{j+1} \right| < \varepsilon_1$ and $\max\limits_{i=0,n} \left| \overset{s+1}{u}{}_i^{j+1} - \overset{s}{u}{}_i^{j+1} \right| < \varepsilon_2$. If these conditions are not satisfied, s is increased by unity, and we return to item b); but if they are satisfied, we go over to the next time layer.

Computational Experiment and Discussion of Results. In the computational experiment we studied the influence of the injected gas temperature and the equations of gas state on the dynamics of the change in the temperature and pressure in the bed in the regime of assigned mass flow rate, i.e., under boundary condition (2). In all of the variants, the parameters c_p/R = 5 and δ = 0.4, initial temperature of the bed of 283.33 K, initial pressure of $100 \cdot 10^5$ Pa, the critical temperature of 200 K and pressure of $45.7 \cdot 10^5$ Pa remained constant, and the gas was consid-

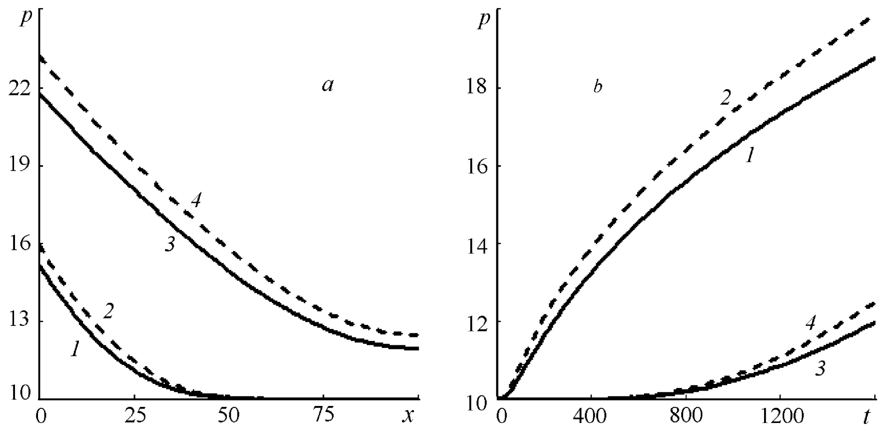

Fig. 2. Distribution (a) and the dynamics of change (b) in the gas pressure at a small flow rate for the second variant of calculation (1, 3) perfect gas; 2, 4) imperfect gas); a: 1, 2) t = 200; 3, 4) t = 1600; b: 1, 2) x = 20 m; 3, 4) x = 100 m. p, MPa; x, m.

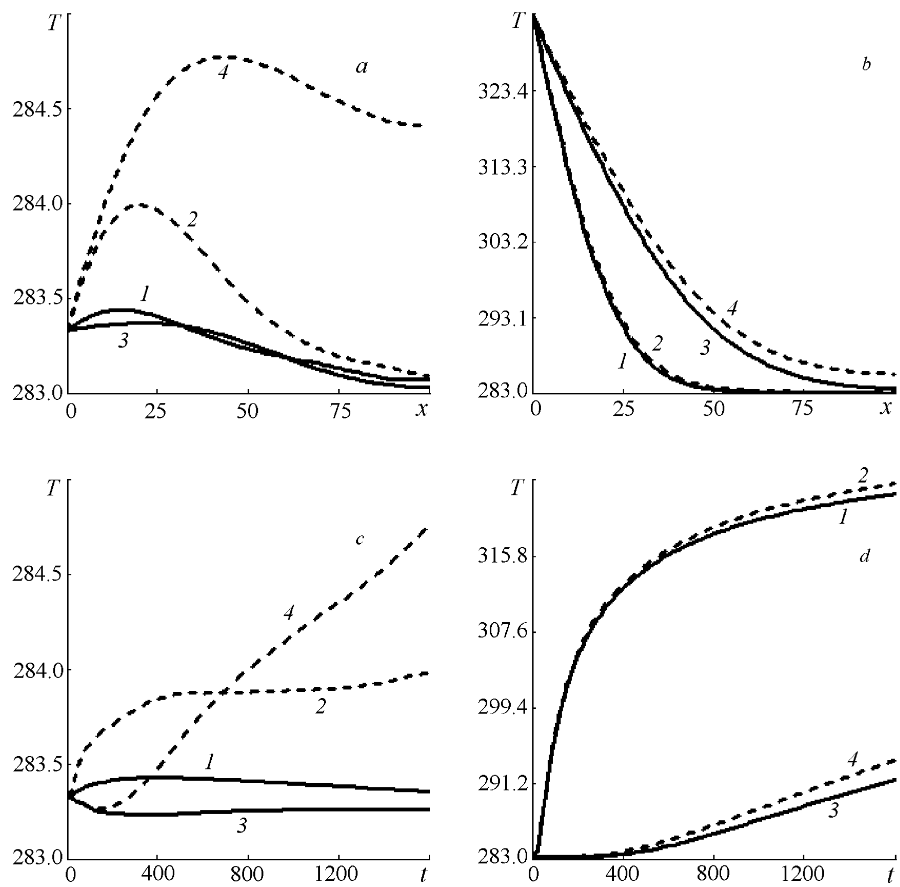

Fig. 3. Distribution (a, b) and the dynamics of a change (c, d) in the gas temperature for the first (a, c) and second (b, d) variants of calculation [1, 3) small flow rate; 2, 4) large flow rate]: a, b: 1, 2) t = 400; 3, 4) t = 1600; c, d: 1, 2) x = 10 m; 3, 4) x = 50 m. T, K; x, m.

ered either perfect (z = 1) or imperfect. Here, the temperature of the injected gas was assumed equal to either the initial temperature of the bed or to 333.33 K. The parameter A (dimensionless mass flow rate) assumed the values 0.00003465 or 0.00016465.

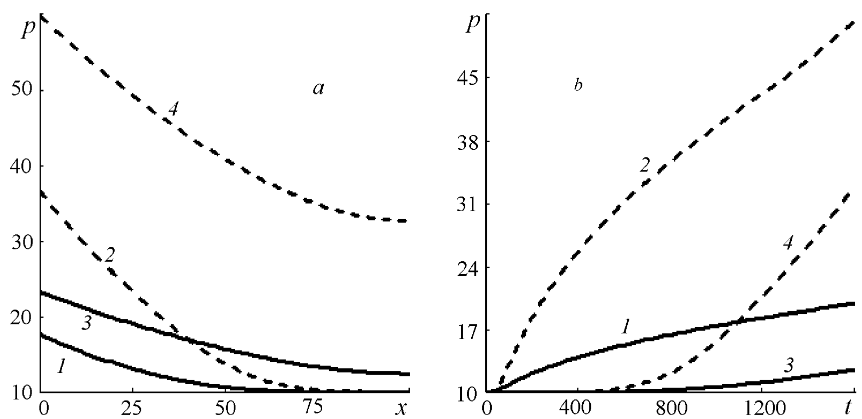

Fig. 4. Distribution (a) and the dynamics of change (b) in the gas pressure for the second variant of calculation [1, 3) small flow rate; 2, 4) large flow rate]: a: 1, 2) $t = 200$; 3, 4) $t = 1600$; b: 1, 2) $x = 20$ m; 3, 4) $x = 100$ m. p, MPa; x, m.

We will analyze the results of the computations presented in Figs. 1–4 (a dimensionless time is used everywhere). First we consider the case of a comparatively small mass flow rate, when the parameter $A = 0.00003465$. If in this case the temperature of the injected gas is equal to the initial temperature of the bed (the 1st variant), then its distribution from the injection to the bed boundary for a perfect gas in the initial period has a wave character (curve 1 in Fig. 1a) and thereafter it monotonically decreases due to the adiabatic expansion of the gas (curve 3 in Fig. 1a), whereas for an imperfect gas the wave character of distribution is preserved during the entire computational time (curves 2 and 4 in Fig. 1a). However, if the ignited gas temperature exceeds the initial one (the 2nd), then from the very beginning it changes monotonically (Fig. 1b), and the character of the corresponding curves points to the prevailing conductive transfer of heat. Interesting features in the temperature variation in time manifest themselves in the 1st variant. First, at a far distance from the injecting gallery (50 m) the temperature of the perfect gas decreases monotonically (curve 3 in Fig. 1c), whereas the temperature of the imperfect gas first decreases, and then, in about 150 days, begins to increase slowly (curve 4 in Fig. 1c). Second, at a distance of 10 m from the injection point the temperature of both gases depends nonmonotonically on time: it increases at the beginning and then begins to decrease (curves 1 and 2 in Fig. 1c). In the second variant the temperatures of both gases are virtually the same (Fig. 2d). The character of the change in pressure for these two variants differs insignificantly, therefore we will give only the results obtained for the second variant. First, the pressure increases sharply at the face of ignition wells: by about a factor of 1.5 in 90 days (curves 1 and 2 in Fig. 2a) and almost twice for the entire period of ignition (curves 3 and 4 in Fig. 2a). At the same time at the bed boundary the pressure increases by about 20% (curves 3 and 4 in Fig. 2b).

Now, we will estimate the influence of the intensity of ignition on the dynamics of the temperature and pressure fields, i.e., we will compare the results of the computations performed for two values of the dimensionless mass flow rate of the gas: 0.00003465 and 0.00016465. We will make a comparison for the imperfect gas. For the 1st variant the corresponding curves are given in Fig. 3a and c, and for the 2nd variant in Fig. 3b and d for the temperature, and in Fig. 4a and b for the pressure. It is seen that the influence of the rate of gas injection on the dynamics of the temperature field is most significant for the 1st variant, when the injected gas temperature is equal to that of the bed (see the corresponding curves in Fig. 3a–d). Here, attention is drawn to the still more clearly expressed nonmonotonicity in the change in the gas temperature in time on increase in the injection rate (curves 2 and 4 in Fig. 3c). We should note, however, that in both cases the gas temperature and the rate of its variation increase with the mass flow rate, although the qualitative characteristics of the temperature field dynamics are preserved in this case. The same tendency is observed in the 2nd variant of calculation too. It is natural that the multiple increase in the mass flow rate of the injected gas leads to a considerable increase of the pressure itself and of the rates of its increase (see the corresponding curves in Fig. 4a and b).

Conclusions. From the results given above it follows that the characteristic features of the thermodynamic processes in the course of imperfect gas filtration are to be studied together with the process of gas transfer by a filtering flow, since only in this case can their interdependence be seen. In the present investigation this was manifested in the effect exerted by heat conduction on the change in temperature due to the Joule–Thomson's throttling effect and adiabatic gas expansion (contraction).

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NOTATION

c_p, specific heat of a gas at constant pressure, J/(kg·K); c_r, bulk heat capacity of the bed saturated with a gas, J/(m^3·K); H, bed thickness, m; h, grid step in space $\overline{\omega}_h = \{x_i = ih,\ i = \overline{0, n}\}$, m; k, coefficient of bed permeability, m^2; l, characteristic dimension, m; m, porosity; M, mass flow rate of a gas, kg/sec; p, pressure, Pa; R, gas constant, J/(kg·K); t, time, sec; T, temperature, K; x, current coordinate, m; z, coefficient of gas imperfection; κ, thermal diffusivity of the bed saturated with a gas, m^2/sec; κ_p, piesoconductivity of the bed saturated with a gas, m^2/sec; μ, dynamic viscosity of a gas, Pa·sec; τ, step of a grid in time $\overline{\omega}_\tau = \{t_j = j\tau,\ j = \overline{1, j_0}\}$, sec. Subscripts: 0, initial state; c, critical; r, rocks; w, on the wall of injection wells.

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